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(54) **Digital filter with parallel analogue path**

(57) An analogue signal filter device which comprises two paths in parallel between input and output, having an analogue filter 18 in one path and an analogue-digital-analogue (A-D-A) filter 20 in the other path, together with a summing means 22 summing the outputs of the two filters to provide the output of the device, whereby distortion introduced by the A-D-A filter is reduced.

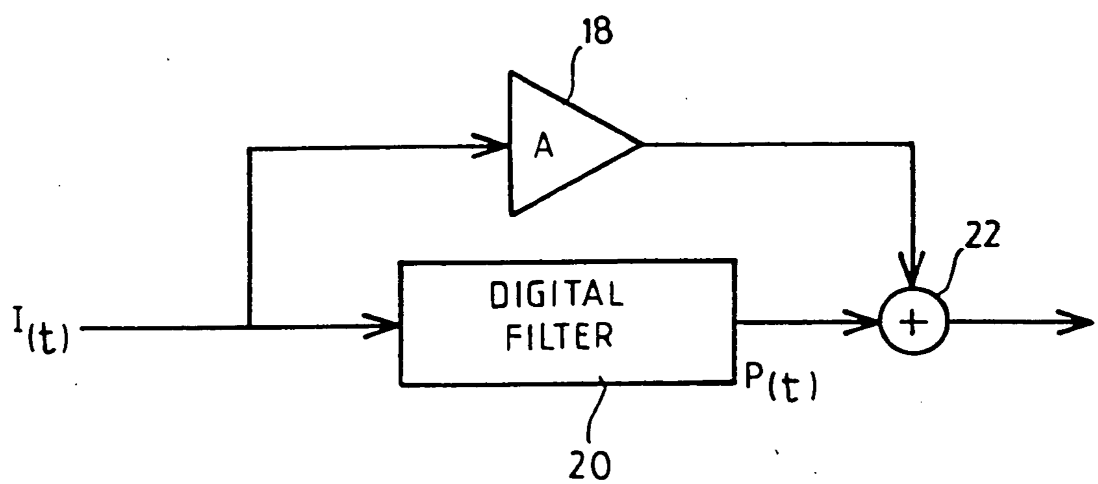

Fig. 2

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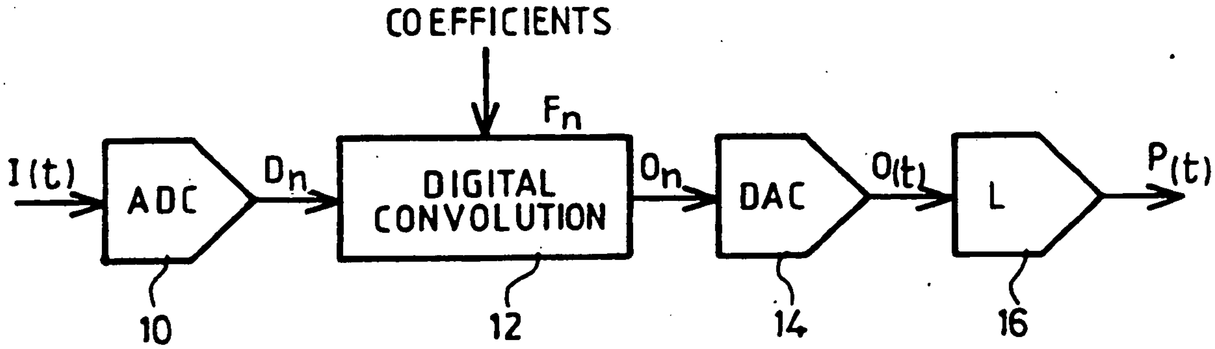

Fig. 1

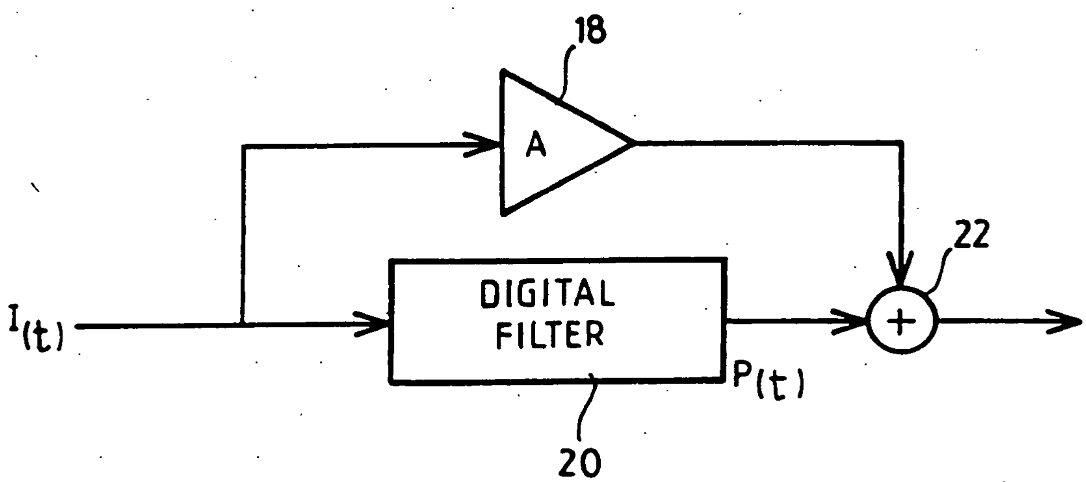

Fig. 2

FORMULA SHEET 1/5

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$$D_n = I(t_o+nT)$$

Equation (1)

$$D(t) = \sum_{i=o}^{\infty} D_i\delta(t-(t_o+iT))$$

Equation (2)

$$F(t) = \sum_{i=o}^{M} F_i\delta(t-iT)$$

Equation (3)

$$O(t) = \sum_{i=o}^{\infty} O_i\delta(t-[t_o+iT])$$

Equation (4)

$$\epsilon^2 = \frac{1}{E_I^2}\int_{-\infty}^{\infty}\left[(FL^+)(FL^+)^* - (FL^+)\left[\frac{L^*G}{L^-}\right]^* - (FL^+)^*\left[\frac{L^*G}{L^-}\right] + G^*G\right]\sigma_I^2 df$$

Equation (9)

Parseval's theorem states that for two functions (a) and (b),

$$\int_{-\infty}^{\infty} a(t)b(t)dt = \int_{-\infty}^{\infty}\bar{a}(f)\bar{b}^*(f)df$$

$$= 0 \text{ if a is causal and b acausal}$$

Equation (10)

$$\int_{-\infty}^{\infty}\bar{a}(f)\bar{b}^*(f)df = \int_{-\infty}^{\infty}\bar{a}(f)\left\{\left[\bar{b}(f)\right]_+^* + \left\{\left[\bar{b}(f)\right]_-^*\right\}df\right.$$

$$= \int_{-\infty}^{\infty}\bar{a}(f)\left\{\left[\bar{b}(f)\right]_+^*\right\}df$$

Equation (11)

$$\epsilon^2 = \frac{1}{E_I^2}\int_{-\infty}^{\infty}\left[(FL^+)(FL^+)^* - (FL^+)\left[\frac{L^*G}{L^-}\right]_+^* - (FL^+)^*\left[\frac{L^*G}{L^-}\right]_+ + G^*G\right]\sigma_I^2 df$$

Equation (12)

$$F_{OPT} = \frac{1}{L^+}\left[\frac{L^*G}{L^-}\right]_+$$

Equation (12A)

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$$\varepsilon_A^2 = \varepsilon^2 - e^2(F)$$

Equation (17)

$$e^2(F) = \varepsilon^2(F)$$

Equation (18)

The error is reduced to zero by the addition of the bypass (18) when (FL-G) is constant for all frequencies at which $\sigma_I^2 \neq 0$

Equation (19)

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Title: Improved digital filter

Field of invention

This invention concerns digital filters particularly such filters for use in active control systems.

Background

A digital filter can be used to filter an analogue signal by sampling the signal, digitising, filtering in digital form and then reconverting to analogue form. The result is an analogue signal which is much wider-band than the input signal owing to the process of reconstruction of the analogue signal. This analogue output is therefore usually filtered by analogue circuitry to limit its bandwidth. The final output is distorted both by the non-ideal digital filter (signal propagation delays etc) and by the in-band effects of the analogue band-limiting filter. These effects can be particularly detrimental in active control applications, where filter response can be critical.

Summary of the invention

In an analogue signal filter two paths are provided between input and output, the one path comprising an analogue-digital-analogue (A-D-A) filter and the other comprising an analogue filter, and the outputs of the A-D-A and analogue filters, are summed to provide the filter output.

By providing an analogue bypass path for the input signal, which is summed with the A-D-A filter output to provide the final output, the effect of the distortion introduced by the A-D-A filter can be reduced. In particular the analogue bypass compensates for delays through the A-D-A filter.

The invention will now be described by way of example with reference to the accompanying drawings, in which

Figure 1 shows a conventional A-D-A filter, and

Figure 2 shows a filter constructed in accordance with the invention.

Figure 1 shows how a digital filter can be adapted to filter an analogue signal. The input is a continuous signal in time, I(t). The ADC (analogue-to-digital converter) 10 samples the signal I(t) at regular intervals of time T, and outputs a number representing its magnitude at each such instant. The output signal D, from the ADC is thus a sampled version of I(t), whoses n'th sample is given by Equation (1) where t_o is the time at which sampling starts.

The series of numbers D_n corresponds to a time series D(t) contained in Equation (2), which has the property $D(f)=D(f+if_N)$ (where i = any integer) and where D(f) is the Fourier Transform of D(t) and $f_N=\frac{1}{2T}$

is the 'Nyquist frequency' of the sampling process (Vide Lynn PA "The analysis and processing of signals." MacMillan 1973. Chapter 4). $\delta(t)$ is the Dirac delta

function.

Thus D(f) has infinite bandwidth.

Digital filtering is accomplished by convolving the series D_n in Convolver 12 with a series of coefficients F_n corresponding to the signal F(t) - see Equation (3) (where M+1 is the order of the digital filter) to produce a new series O_n corresponding to a time series O(t) whose fourier transform is given by $\overline{O(f)} = \overline{F(f)} x \overline{D(f)}$ (where a bar superscript again indicates the Fourier Transform of the corresponding time signal).

The digital-to-analogue converter ("DAC") 14 produces an analogue output signal proportional to each element O_n of O as it is produced. Ideally each element O_n is converted precisely at the instant that the corresponding I_n is produced by the ADC and is given by:-

$$O_n = I_n F_o + I_{(n-1)} F_1 + \ldots\ldots + I_{(n-M)} F_M$$

Ideally all conversions are instantaneous and perfect, so that ADC 10, Convolver 12 and DAC 14 produce corresponding output values at the same instant. The ideal resultant analogue signal is then give by Equation (4).

As with the signal D(t), the signal O(t) has infinite bandwidth. This signal must therefore be band-limited using an analogue filter L to remove unwanted frequency components produced by the sampling process.

In practise this effect is often achieved by causing the DAC to hold its output constant at the value of the last sample O_n until O_{n+1} is received at its input. This is

equivalent to L having the impulse-response:

$L(t) = 0.$ for t less than 0,
$L(t) = 1.$ for t in the range 0 to T,
$L(t) = 0.$ for t greater than T.

In practise also, the combination of ADC 10, digital Convolver 12 and DAC 14 introduces propagation delays into the signal. These will be considered as well as any hold period of the DAC, as part of the equivalent output filter 16.

It is first necessary to consider the effect of the filter 16 on the signal in the passband. Suppose that we wish to implement the equivalent of an analogue filter with impulse response G(t) using the A-D-A filter. Restricting ourselves to $|f|$ less than f_N the Convolver would ideally be programmed with coefficients F_n, equivalent to the signal F(t), where F(t) and G(t) are linked by Equation (5).

In practise, this may require that F(t) has an anticipatory (acausal) part. The design criterion can be generalised to find the F(t) with no acausal part which best satisfies Equation (5) within the band of the input signal I(t), using the L_2 (least-squares) norm as our measure. We define the square of the error as by Equation (6).

In Equation (6) the second squared term in the integral and is the variance of the input signal at frequency f and E_I^2 is given by Equation (7).

We require to find F(t) such that the left-hand side of

Equation (6) is minimised. Dropping arguments and bar superscripts for brevity, Equation (6) can be re-written as shown in Equation (8).

In Equation (8) the superscript '*' indicates a complex conjugate.

F is constrained to be causal.

It is now necessary to replace L in the term $(FL)^*$ by the causal minimum-phase signal L^+, for which $LL^* = L^-L^+$ where $L^- = (L^+)^*$.

If L corresponds to a physically realisable filter, then a signal (L^+) can always be found.

Thus we may rewrite Equation (8) as Equation (9).

The product FL^+ is known to represent a causal function. We can therefore apply Parseval's theorem (see Equation 10) to Equation (9) with FL^+ corresponding to (a), and $\left[\frac{L^*G}{L^-}\right]$ corresponding to (b). We also split (b) up into the sum of its causal and acausal parts: $b = [b]_+ + [b]$

where $[b]_+$ is the causal part of b
$[b]_-$ is the acausal part.

Parseval's theorem then allows us to rewrite this as shown in Equation (11) and Equation (6) can be rewritten as Equation (12).

In Equation (12) all terms involving $a[b]_-$ (whose integral equals zero) have been dropped.

by inspection, this can be minimised with respect to F By setting F equal to the value given by Equation (12A).

The minimum error value is then given by Equation (13).

Since any increase in delay in L will tend to make $\frac{L^{*}G}{L^{-}}$ more acausal, it may increase the minimum error in implementing G. This is particularly likely to be so when the delay in G itself is small, ie if G(t) is largest near t=0. The technique of implementing L by causing the DAC to hold its output value over time T introduces a delay of $\frac{T}{2}$ into the output signal, and so can be a cause of this form of error.

In Figure 2, an analogue bypass 18 is incorporated in accordance with the invention, and is implemented by amplifying the input signal I(t) by some scalar A and adding the result in adding stage 22 to the output signal P(t). Its use will almost always enable any degradation in performance produced by 16 in the digital filter 20, to be reduced.

The L_2 norm error of implementation, using the analogue bypass is given by Equation (14) for any given F where the first term in the third expression is as defined in the previous section (ie. is the error with no bypass). This can be written as shown in Equation (15).

The function e is a functional of function F(f).

It is defined in equation 16.

By setting A=-e(F) we achieve a new minimum value equation (15), given by Equation (17)

This will represent no improvement only if the left-hand side of Equation 9 = 0 or e = 0. The first will only be true if L is <u>minimum phase</u> and G is causal, when $F_{OPT} = \frac{G}{L}$, (since $L = L^+$ and $L^* = L^-$ for a minimum phase function)

If Equation (9) is not equal to zero, then it may still be true that e(F)=0, and in this case there is obviously no improvement to be gained from including the analogue filter 18.

The error is reduced to zero by the addition of the bypass when F(f) is such that Equation (18) is satisfied.

The error of fit to G in the time domain is then a spike at t=0 and effectively the bypass provides the first (t=0) coefficient of G_n, and the digital filter programmed with F_n provides the remainder. This will not be exactly achievable in practise because (FL-G) must be a causal band-limited function and so cannot correspond to a spike at t = 0.

The condition that Equation (18) is satisfied is given by Equation (19).

CLAIMS

1. An analogue signal filter device having two paths between input and output, one path comprising an analogue-digital-analogue (A-D-A) filter and the other path comprising an analogue filter, means being provided for summing the outputs of the A-D-A and analogue filters to provide the output of the device.

2. A device according to claim 1, wherein in the A-D-A filter, the leading analogue filter samples the input signal at regular time intervals, digital filtering is accomplished by convolving the series of signal outputs produced by the sampling with a series of coefficient representative of a predetermined signal, thereby to produce a filtered series of signal outputs, and the trailing analogue filter produces an analogue signal proportional to each output of the filtered series.

3. A device according to claim 2, wherein the predetermined signal is a signal F(t) in accordance with Equation (3) hereinbefore defined, where (M+1) is the order of the digital filter.

4. A device according to claim 2 or claim 3, wherein the trailing analogue filter is bandwidth limited.

5. A device according to claim 4, wherein bandwidth limiting is achieved by holding the last filtered sample O_n until the filtered sample O_{n+1} is received at its

input.

6. A device according to claim 5, wherein delays introduced by the digital filtering and following analogue conversion are compensated for by selecting non-acausal coefficients for the predetermined signal so that said signal best satisfies equation (5) hereinbefore defined, within the band of the input signal, using a least squares measuring technique.

7. An analogue signal filter device substantially as hereinbefore described with reference to the accompanying drawings.

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